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RISK MODELING IN QUALITY ASSESSMENT OF READY-MIX CONCRETE USING SIMULATION METHODS

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Abstract
Uncertainties associated with assessing the strength of concrete and the shortcomings of statistical criteria for assessing compliance are the reasons for formulating new procedures for classifying manufactured concrete, as well as for estimating risk when assessing its quality. Using the proposed model based on simulation methods and Gauss or Clayton copulas, it is possible to calculate the value of risk in the production of ready-mixed concrete with underestimated quality. Exposure to risk in assessing the quality of concrete can be measured as the product of the level of risk and the consequences of embedding under-quality concrete in a structure, and the developed matrix makes it possible to determine the level of risk and contribute not only to its quantification during the various phases of its production, but can affect the final quality of the concrete produced. Using the proposed model, the article calculates the level of risk in the production of ready-mixed concrete with underperformance.

Keywords: **Risk; Ready-mix concrete; Fuzzy sets; MC simulations; Copulas functions.**

1. INTRODUCTION

Understanding and effectively managing risks in the production of ready-mix concrete can not only contribute to better management of risks, during the various phases of its production, but can affect the final quality of the concrete produced. Risk management can effectively influence decision-making and implementation of actions leading to an acceptable level of risk. Concrete, as a structural building material, is subject to conformity and identity assessment according to the recommendations of PN-EN 206 [1] and its national supplement PN-B-06265 [2]. However, the standard evaluation methods are not perfect, because they were developed with the assumption of simplifications, which result in the fact that the standard criteria take into account only the risk of the producer, completely ignoring the risk of the concrete user [3, 4, 5]. Producers choosing cost-optimal procedures, resulting in the selection of the cheapest evaluation method, do not analyze the impact of the choice of evaluation criterion on the risk of the customer, investor, or contractor of the object. So far, the analyses carried out mainly concern the form of criteria

[3, 4], and only a few works deal with statistical-fuzzy evaluation of the quality of produced concrete [4, 5], or risk analysis, for example, using fuzzy sets [6] and simulation methods, so this paper undertakes risk estimation using simulation and matrix methods. It should be emphasized that effective assessment of quality, [7, 8, 9, 10, 11] reliability [12–15], safety [16–17] or risk [18] of both objects , and /construction processes enable prospective validation of construction investments.

2. METHODS

An algorithm and an example using a fuzzy logic system for calculating the value of risk in the production of underperforming ready-mix concrete was presented at Krynica 2022 [6]. The fuzzy logic system included membership functions, inference rules and three defuzzification methods. Another approach is the use of simulation methods and Gauss and Clayton copulas [19–24].

2.1. Copulas functions

Copula functions are primarily used to analyze multi-dimensional distributions. Their idea is to represent a multi-dimensional distribution by two parts: boundary distributions and joint functions [21]. The most commonly used method for estimating copula parameters is the method of maximum credibility estimation. To use this method, it is necessary to know the parameters of the boundary distributions, which in practice means that they must be estimated by other methods. Therefore, in the literature there is another way of estimating the parameters of copulas, the so-called inference for boundary distributions. This method is based on the observation that there are two components in the logarithmic credibility function, with one containing only the parameters of the boundary distributions, and the other additionally containing the parameters of the copulas as well. Estimation is then carried out in two steps. In the first, the parameters of the marginal distributions are estimated by selecting them so as to maximize the first component. In the second step, the copula parameters are selected to maximize the second component with the determined estimators from the first step. Both procedures are time-consuming in practice. In the case of a copula depending on a single parameter, a simpler way of estimating this parameter can be given. It consists in determining an estimate of Kendall's τ coefficient (or ρs - Spearman's) from the data. In the next step of the procedure, the determination of the copula parameter can be carried out according to the procedure discussed in detail in [22, 23].

Many examples of copulas are given in the literature [21–24]. In the theoretical study, the results of which are presented later in this article, two linkage functions were used:

- Gauss copula:

$$C(u,v,\theta) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(\frac{-(s^2-\theta\cdot st+t^2)}{2(1-\theta^2)}\right) dsdt$$

where: ϕ is the distribution of the standard normal distribution and $\theta \in (-1,1)$.

- Clayton copula

$$C(u,v,\theta) = \left(u^{-\theta}+v^{-\theta}-1\right)^{\frac{1}{\theta}}, \theta\epsilon(0,+\infty)$$

In the case under consideration, the choice of both boundary functions and copulas was made arbitrarily. In the case of the Clayton copula, the advantage is the uncomplicated analytical form [23].The iteration procedure in the Monte Carlo simulation for the Gauss and Clayton copulas is discussed in detail in [23].

3. CASE STUDY

The theoretical analyses described in this section are intended to demonstrate the applicability of Gauss and Clayton copulas for estimating risk values in ready-mix concrete production. The level of risk was evaluated, taking into account the complexity of the decision on the actual quality of the produced batches of ready-mixed concrete of class C16/20 verified on the basis of a sample size of n = 3 and the defectiveness during compliance inspection – w_1 and defectiveness after inspection – w_2. In the example under consideration, the boundary distributions of the input variables were defined through the use of simulation methods and arbitrarily, while the dependency function for the *Value of Risk* was defined on the basis of analyses conducted using fuzzy logic (the research was presented at the Krynica 2022 Conference) [6]:

$$Value\ of\ Risk = 0{,}1\cdot f_{cm} + 0{,}2\cdot w_1 + 0{,}7\cdot w_2 \tag{1}$$

To determine the density function of compressive strength, a statistical-fuzzy approach was used to determine the boundary distribution for the considered class of C16/20 concrete [21, 22]. This method was proposed by Yager [25], and used for statistical-fuzzy classification of concrete by Sz. Wolinski [10], among others. The first step is to determine the parameters of the density function, for the defined random variables ξ and η. The variable ξ represents the separation point of compressive strength values for the considered and neighboring-lower class of concrete, and η represents the separation point of the considered and neighboring-higher class of concrete. It was assumed that the pair (ξ, η) is a two-dimensional normal random variable, for which the marginal distributions $p_\xi(\mathrm{k})$ and $p_\eta(\mathrm{k})$ of the random variables $\xi \rightarrow N(m_\xi, \sigma_\xi)$ and $\eta \rightarrow N(m_\eta, \sigma_\eta)$ can be determined according to formulas (2) to (5). According to the calculation algorithm, the boundary distribution function of the compressive strength for the considered and lower class of concrete can be described by formulas (4) and (5) [10]:

$$\mu_{Ci}(f_{cm}) = 1 - \int_{f_{cm}}^{+\infty} p_\xi(f_{cm})\, df_{cm} - \int_{-\infty}^{x_n} p_\eta f(f_{cm}) \tag{2}$$

$$\mu_{Ci}(f_{cm}) = 1 - \left[1 - F\left(\frac{f_{cm} - m_\xi}{\sigma_\xi}\right)\right] - F\left(\frac{f_{cm} - m_\eta}{\sigma_\eta}\right) \tag{3}$$

$$\mu_{Ci}(f_{cm}) = F\left(\frac{f_{cm} - m_\xi}{\sigma_\xi}\right) - F\left(\frac{f_{cm} - m_\eta}{\sigma_\eta}\right) \tag{4}$$

where:

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} exp(-0{,}5z^2)dz \tag{5}$$

The three-phase method (fuzzy-statistical) has been used to determine membership functions of codes conformity criteria for compressive strength [1]:

$$f_{cm} \geq f_{ck} + 4 \tag{6}$$

$$f_{ci} \geq f_{ck} - 4 \tag{7}$$

where: f_{ck} – characteristic compressive strength of concrete, f_{cm} – mean compressive strength in the test, f_{ci} – minimum compressive strength in the test.

To generate random numbers with standard normal distribution was used by the MC method. Built an array of probability distribution function of random vector (ξ, η) and the histogram of distributions for the boundary determined by adding the rows and columns: the first (sum of lines) – classified by the considered and lower class of concrete, the second (the sum of columns) were classified by the considered and higher class of concrete. Graphs of the density function of boundary probability distributions $p_\xi(f_{cm})$ and $p_\eta(f_{cm})$ are the basis for the designation of membership function test characteristics for each class of concrete. On the basis of simulations for the concrete class C16/20, generating 100 000 random groups of size n = 3 in accordance with normal distribution were estimated marginal density functions of distributions and fuzzy membership functions for considered class of concrete. The marginal probability distributions $p_\xi(x)$, $p_\eta(x)$ and the plot of original and membership function of concrete C16/20 are presented by the Fig. 1.

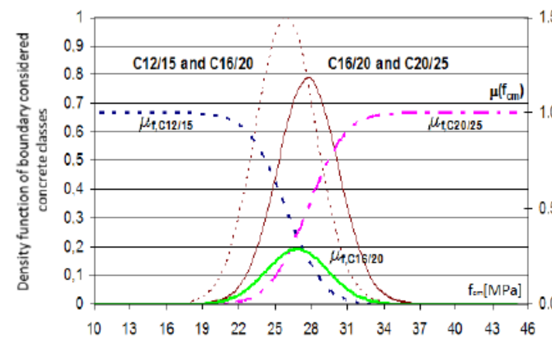

Figure 1.
The membership function of f_{cm} for considerate concrete class C16/20 and neighboring classes C12/15 and C20/25

The density functions for defectiveness during inspection and after inspection were assumed to be Gamma and linear functions [5]. Having the marginal distributions for the considered input data, assuming that the input variables are correlated, and the joint distribution is modeled with selected types of copulas, the risk distribution was determined using a simulation method. Risk estimation includes the following steps:

- in the first step, the combined distribution of $CD(D_{fcm}, D_{w1}, D_{w2})$ is modeled by using individual types of copulas with defined boundary distributions D_{fcm}, D_{w1}, D_{w2} (8):

$$CD(D_{fcm}, D_{w1}, D_{w2}) = C(D_{fcm}, D_{w1}, D_{w2}) \tag{8}$$

where: $CD(D_{fcm}, D_{w1}, D_{w2})$ – combined distribution, $C(D_{fcm}, D_{w1}, D_{w2})$ – copulas with boundary distributions of D_{fcm}, D_{w1}, D_{w2}

- next, for the function *Value of Risk* (9):

$$Value\ of\ Risk(i) = 0{,}1 \cdot f_{cm}i + 0{,}2 \cdot w_1 i + 0{,}7 \cdot w_2 i + 4{,}0 \tag{9}$$

Values $(f_{cm}i,\ w_1 i,\ w_2 i) \sim CD(D_{fcm}, D_{w1}, D_{w2}) = C(D_{fcm}, D_{w1}, D_{w2})$ are generated by the Monte Carlo method from a given copula.

In the case of the Gaussian copula [19, 20], guided by

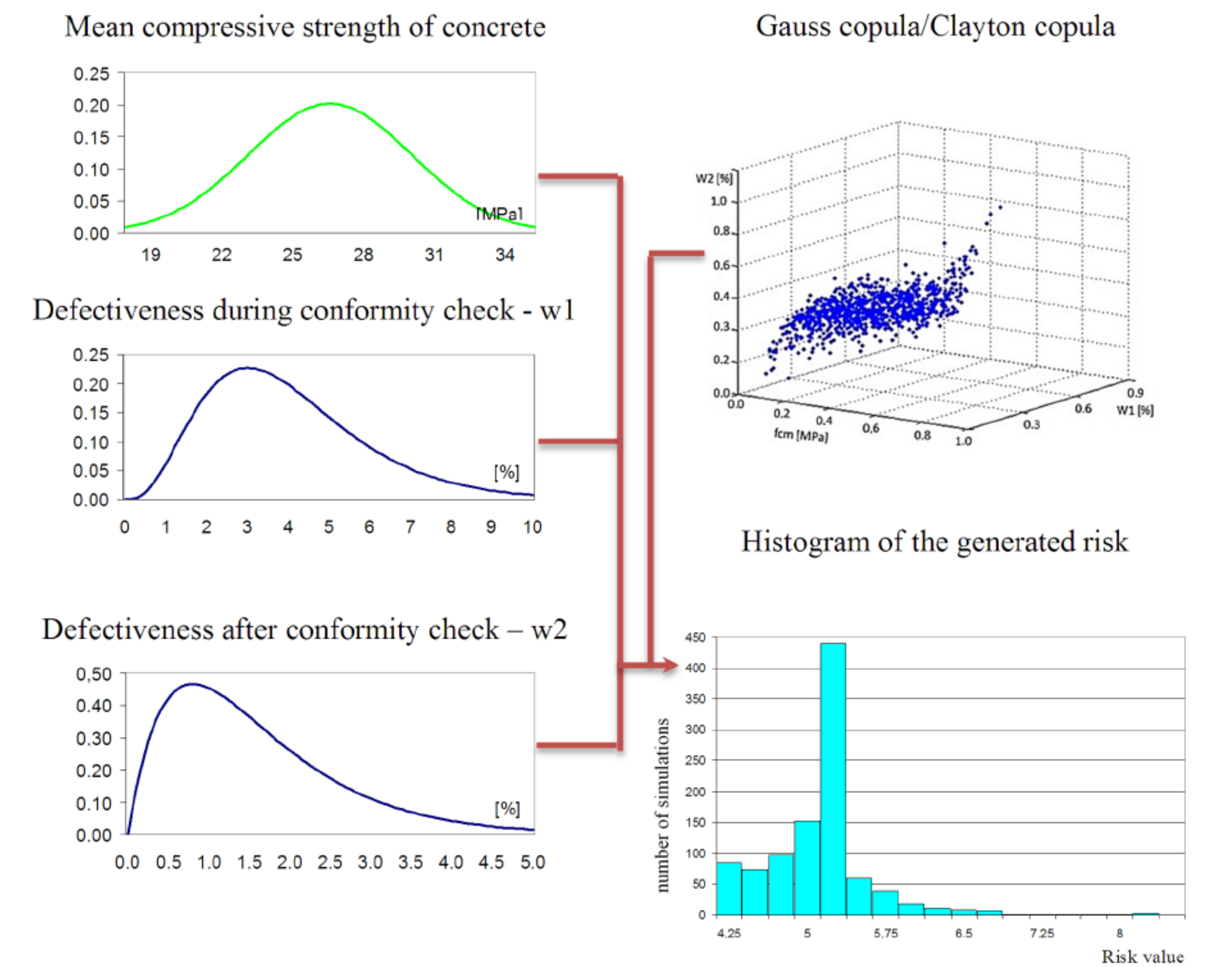

Figure 2.
Block scheme – simulation of the risk output variable (scheme developed based on [7])

Taerwe's research on autocorrelation of compressive strength [11], the correlation matrix of individual variables was defined, assuming $\rho = 0.8$ for all variables; in the case of using the Clayton copula [19, 20], a strong positive correlation was also assumed, assuming $\theta = 6$. Using the simulation method and the selected copulas, the risk distribution of improper quality assessment of ready-mix concrete was estimated (Fig. 1) and basic descriptive statistics of the risk were calculated. In the adopted model, the input values are the controlled parameters, i.e. the average compressive strength of concrete for a sample size of $n = 3$, defectiveness during compliance inspection and defectiveness after inspection. The defined parameters determine the occurrence of underperforming concrete and are the input parameters, while the risk value is the output. Calculations were performed according to the formulas given in [19] and the following scheme developed also according to [19].

Risk estimation was performed according to the initial assumptions defined in Section 2 and 3. In order to test the convergence of the estimated risk values using the simulation method, initial calculations were performed by generating N groups of random numbers with a count of $n = 100{,}000$ (Table 1) using an Excel.

Table 1.
Risk values estimated for different types of copulas and different sample sizes

Type of cupola	Number of random groups N	Value of risk
Gaussa	1000	5.70
	100000	5.57
Claytona	1000	5.91
	100000	5.85

Since the relative differences in the results obtained for random groups of 100,000 and 1,000 were 2.33%

for the Gauss copula and 1.02% for the Clayton copula, respectively, in further risk analyses, we were limited to generating N = 1,000 random number groups of n = 100,000 from the probability distribution density function of each random variable. The response sets obtained from the simulations were subjected to statistical evaluation. Using the proposed calculation model, the estimated level of risk in the production of under-quality ready-mixed concrete is (Table 2).

Table 2.
Simulated risk of misconduct

Parameter	Gauss copula	Clayton copula
Number of simulations	1000	1000
Maximum	7.30	8.27
Minimum	0.21	4.
Mean	4.46	5.11
95% *VofR*	5.70	5.91

In order to interpret the estimated risk values, they were compared with the values defined for the risk matrix developed in [6]. The risk matrix taking into account the parameters adopted for the analysis is presented in the form of Table 2. The values in Table 2 were determined using formula (10):

$$R = \sum_{i-1}^{n} p(E_i) \cdot D_i \qquad (10)$$

where: E_i – point weight regarding the probability of occurrence of the considered parameter/event, D_i – the point weight that determines the losses associated with the occurrence of this incident.

According to the matrix (Table 3), risk is described by a three-level scale: low risk R_L = [1÷2], medium risk R_M = [3÷5], high risk R_H= [6÷9].

Table 3.
Risk matrix calculated according to formula (9)

E_i	D_i		
	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9

According to the defined matrix, the obtained values indicate that the estimated risk of a produced batch of concrete with underperformance is high. In practical issues, the simulation stage must be preceded by estimation of the unknown parameters of the copulas on the basis of empirical data.

3. CONCLUSIONS

Uncertainties associated with the assessment of concrete strength and the defects of statistical compliance criteria are the reasons for developing risk estimation procedures during the quality control of manufactured concrete. Currently, the concept of control class (level) is discussed quite widely. However, tighter practical recommendations in this regard are still lacking, since different classes of inspection do not mean different levels of quality, but only different levels of reliability. According to these levels, all requirements that characterize the quality of the entire structure or its individual components are controlled. In this way, it is possible to speak of accepting lower and higher inspection requirements due to the assumed reliability of the structure. In practice, different classes of control are used depending on the risk of danger to human life and the consequences of destruction. Raising the inspection class usually means increasing the frequency of inspections, increasing the number of samples for inspection tests or increasing the scope of tests. Using the proposed model based on simulation methods and Gauss and Clayton copulas, it is possible to calculate the value of risk in the production of ready-mixed concrete with underestimated quality. Risk exposure in the evaluation of concrete quality can be measured as the product of the level of risk and the consequences of the incorporation of under-quality concrete into a structure. The developed matrix and the procedure for estimating risk by simulation methods makes it possible not only to determine the level of risk, but can contribute to the quantification of risk during the various phases of its production, and also affect the final quality of the concrete produced. Gauss and Clayton copulas were used to determine risk. Simulations were carried out for a sample size of 1000, and the risk value was taken as the 95% quantile of the obtained Vaule of Risk distribution. The risk values obtained using the Gauss and Clayton copulas coincide and are respectively: 5.70 and 5.91, and the estimated risk can be defined as high. The recommendations in the current standards take into account only the manufacturer's risk. Recipient risk remains unspecified. The principle of level playing field suggests that conformity assessment criteria should take into account the rational and informed sharing of risk. The European Union's standardization directive allows the requirements in the standards to be treated as minimum requirements. A concrete customer can agree with a supplier on terms that allow for informed risk selection. One possible strategy is to balance the risks of the customer and the producer. Agreeing on the acceptable risks of the producer and the concrete recipient, allows estimating the probability of confirming compliance and selecting an appropriate control plan. It should be noted that in terms of risk assessment, there may be overestimation or underestimation of risk. Overestimation of risk may be beneficial in terms of safety of facilities and their users, but may result in overestimation of the cost of prevention. Failure to overestimate can lead to an increase in the cost of repairs, renovations, while the cost of prevention is reduced. The authors' proposed method of estimating risk using Gauss and Clayton copulas would need to be analyzed in relation to empirical data. At this stage of the research, the study conducted using copulas is only a theoretical analysis. The analyses described in the article are a prelude to further research.

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